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During a workshop held to improve primary and intermediate arithmetic programs for the academically gifted children in the Cincinnati schools, the teacher participants developed a series of ideas and exercises. The material is intended to provide challenging enrichment experiences for many children and to be of help to administrators and teachers in other school systems. Among the concepts explored and presented are: numeration system - prime numbers, factors, exponents and other bases for numeration; operations - addition, subtraction, multiplication, division and more than one operation; and measurement, patterns, inequalities, sets, exploratory geometry, fractions, decimals and per cent. Suggested activities intended to reinforce learning accompany each concept discussed.

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Enrichment Ideas

for

Grades 4 5 6

Issued by E. E. Holt
Superintendent of Public Instruction

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Columbus, Ohio

1962

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ARITHMETIC ENRICHMENT IDEAS

For Grades 4, 5 and 6

Prepared by

Cincinnati Public Schools
Department of Instruction

in Cooperation with
Programs for the Academically Gifted

Under the Direction of
R. A. Horn
Director, Division of Special Education
State Department of Education
Columbus, Ohio

September 1962

FOREWORD

This booklet contains ideas and exercises developed by teacher participants in an Arithmetic Workshop at the University of Cincinnati. The workshop was held to improve the primary and intermediate arithmetic programs for the academically gifted children in the Cincinnati Public Schools.

The accelerated mathematics program in Cincinnati is one of the research projects made possible by state funds through the Program for the Academically Gifted. The State Department of Education is pleased to work cooperatively with the school districts for the further improvement of instruction.

The material presented in this booklet will provide challenging enrichment experiences for many primary children. It is my hope that this material will be of value to administrators and teachers in other school systems throughout Ohio.

E. E. Holt
Superintendent of Public Instruction

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NUMERATION SYSTEM

Our numeration system is a positional system based on groupings of ten.

Prime Numbers

A prime number is a natural number that has only two different factors, itself and one. A natural number that has factors other than itself and one is a composite number.

The Sieve of Eratosthenes

More than 2,000 years ago a Greek named Eratosthenes discovered a way of finding prime numbers. It is said that on a piece of parchment he wrote the natural numbers in order to a certain number. He punched out all the numbers that are not primes. He punched out 1, then all multiples of 2 except 2 itself, then all multiples of 3 except 3, and so on until only prime numbers were left. When he finished his parchment was full of holes. This is called the "Sieve of Eratosthenes."

Activity A

To sift out numbers as Eratosthenes did, list the numbers from 1 to 100 as follows:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

a. Cross out 1 because 1 is not considered prime.
b. Cross out all even numbers except 2.
c. Cross out all the multiples of 3 except 3.
d. Cross out all multiples of 5 except 5.
e. Cross out all multiples of 7 except 7.

Numbers not crossed out are the prime numbers less than a hundred.

Activity B

1. Answer these questions:

a. What odd numbers less than 30 are composite numbers?
b. What number is neither prime nor composite?

c. What is the only even prime number?
d. Is 24 a prime number? Why?
e. Is 2 a composite number? Why?
f. Are there more prime or more composite numbers in the set of whole numbers from 1 through 50?
g. What is the largest prime number that we need to use as trial divisor to see if 149 is prime? Is 149 a prime number?

2. Test the numbers below and list those that are prime numbers.

a. 101 c. 151 e. 613
b. 202 d. 263

Activity C

A prime pair is a pair of consecutive prime numbers with only one composite number between them.

Example: 3, 5

1. What prime pairs can you find less than 100?

2. What two prime numbers have each of these even numbers for their sum?

a. 10 c. 20
b. 56 d. 48

Goldbach's Conjecture

Christian Goldbach (1690-1764) was a Russian mathematician who first noted that:

Every even number except the number 2 can be written as the sum of two primes.

Example:

Even		Prime	+	Prime
6	=	3	+	3
8	=	3	+	5

Activity A

Write thes numbers as the sum of two primes.

	Even		Prime	+	Prime		Even		Prime	+	Prime
a.	18	=		+		f.	48	=		+	
b.	30	=		+		g.	50	=		+	
c.	32	=		+		h.	52	=		+	
d.	26	=		+		i.	54	=		+	
e.	28	=		+		j.	56	=		+	

Activity B

Every odd number larger than 7 can be expressed as the sum of three primes.

Example: 29 = 13 + 13 + 3
Express these odd numbers as the sum of 3 primes.

a. 31 =
b. 11 =
c. 17 =
d. 23 =
e. 29 =
f. 31 =
g. 53 =
h. 61 =
i. 91 =

Activity C

It has been noted but not proved sufficiently that every even number greater than 6 can be written as a sum of 4 primes.

Example: 18 = 2 + 2 + 3 + 11
Write these even numbers as the sum of four primes.

a. 14 =
b. 10 =
c. 28 =
d. 30 =

Patterns Using Primes

Magic squares can illustrate patterns using primes.

Activity A

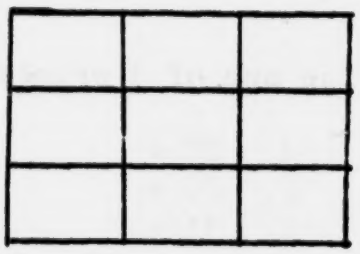
1. Find the sum of the square array of numbers for every row, column, and diagonal.

67	1	43
13	37	61
31	73	7

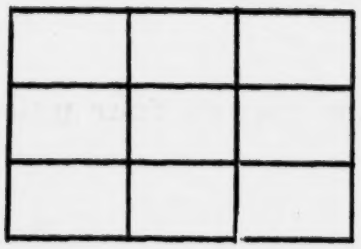
Is this a magic square? Why? How does this magic square differ from other magic squares?
NOTE: 1 is neither prime nor composite.

2. Multiply the number in each cell of the given square by 5. Is the resulting square a magic square? Prove your answer. Is the resulting square a magic square of primes?

3. Subtract 1 from each cell of the original magic square. Is the resulting square a magic square? Prove your answer. Is the resulting square a magic square of primes?



4. Add 44 to each cell of the original magic square. Is the resulting square a magic square? Prove your answer. Is the resulting square a magic square of primes?



Activity B

1. Study this square.

3	71	5	23
53*	11	37	1*
17*	13	41	31*
29	7	19	47

2. Answer these questions.

a. What is the sum of the 4 corners of the square?
b. What is the sum of the four central squares?
c. What is the sum of the shaded squares?
d. What is the sum of the starred (*) squares?
e. What are the sums of each row, column and diagonal?
f. What are some characteristics of this particular magic square?
g. What number in this magic square is neither prime nor composite?

Factors

Factors are the multiplicand and the multiplier of a multiplication problem.

Prime Factors

Every number can be expressed as a product of whole number factors.

Examples:

12 = 1 x 3 x 4; 3 x 4; 2 x 6; etc.
18 = 2 x 9; 6 x 3 ; etc.
30 = 10 x 3; 2 x 3 x 5; etc.

Each composite factor can be expressed as a product of prime factors. Therefore, every number is either prime or can be expressed as a product of prime factors.

Example of whole number factors:
$12 = 3 \times 4$

Example of prime number factors:
$12 = 3 \times 2 \times 2$

Activity A

Express the following numbers as products of prime number factors.

a. $18 = 2 \times 9$; $18 =$
b. $24 =$
c. $20 =$
d. $27 =$

Activity B

Any number can be resolved into prime factors by dividing by its consecutive prime factors, beginning with the smallest.

1. Separate the number 48 into its prime factors.

   a. Divide 48 by 2.

   $$2\overline{)48}\quad 24$$

   b. Continue dividing the quotient by 2 until it is no longer divisible by 2.

   $$2\overline{)24}\quad 12$$
   $$2\overline{)12}\quad 6$$
   $$2\overline{)6}\quad 3$$

   c. Next, divide the quotient by 3.

   $$3\overline{)3}\quad 1$$

2. For other numbers divide the quotient by 3 until it is no longer divisible by 3. Then divide the quotient by 5, 7, 11, etc. so long as division may be done.

3. Find the prime factors of the following numbers:

   a. 85
   b. 121
   c. 413
   d. 2100

Greatest Common Factor

The greatest common factor (G.C.F.) is the largest factor common to two numbers; it is the product of the common factors of the two numbers. It is also known as the greatest common divisor (G.C.D.).

Example:

Find the greatest common factor of 48 and 72.

Prime factors of 48 = 2 x 2 x 2 x 2 x 3
Prime factors of 72 = 2 x 2 x 2 x 3 x 3

Common factors of 48 and 72 are:

2 x 2 x 2 x 3

Therefore, the G.C.F. of 48 and 72 are the products of the common factors.

2 x 2 x 2 x 3 = 24

Activity

1. Gind the G.C.F. of:
   a. 210 and 108
   b. 126 and 105
   c. 308 and 420
   d. 36 and 42 and 60

2. Complete the following exercises by filling in the missing prime factors.
   a. 2 x 2 x □ x 11 = 220
   b. 3 x 5 x △ x 13 = 1365
   c. 3 x 3 x 7 x ▱ = 798

3. Find the G.C.F. of the following set of numbers which have been broken down into their prime numbers:
   a. 540 = 2 x 2 x 3 x 3 x 3 x 5
   b. 420 = 2 x 2 x 3 x 5 x 7
   c. 765 = 3 x 3 x 5 x 17
   d. G.C.D. = □ x □

Common Multiple

A common multiple of two or more numbers contains each of the numbers as factors.

Activity

1. Find the common multiples of 2 and 7 from the numbers 1 through 30. Circle all the multiples of 2 and enclose the multiples of 7 in a square. Thus -

1	(2)	3	(4)	5	(6)	[7]	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

2. Finish this exercise.

3. Answer these questions.
   a. Are there any numbers that have both a circle and square around them?
   b. What can we say about these numbers?
   (These numbers are called common multiples of 2 and 7.)

Lowest Common Multiple

The lowest common multiple (L.C.M.) is the smallest number which will exactly contain each of two or more numbers or the smallest number that has each of the other numbers as factors.

Activity

1. Find the lowest common multiple of 6 and 12.
   Example: Prime factors of 6 are 2, 3.
   Prime factors of 12 are 2, 2, 3.

To find the L.C.M. find the product of the prime factors, using each factor the greatest number of times that it occurs in any one number.

L.C.M. = 2 x 2 x 3 = 12

2. Find the L.C.M. of 6 and 8.

   Prime factors of 6 are _____ and _____.
   Prime factors of 8 are _____, _____, _____.
   The L.C.M. is the product of _____, _____, _____, and _____.

3. Find the L.C.M. of 18, 42 and 60.

   Prime factors of 18 are _____.
   Prime factors of 42 are _____.
   Prime factors of 60 are _____.

   The L.C.M. is the product of _____, _____, _____, _____, _____, and _____.

4. Find the L.C.M. of 12, 15, and 30.

5. Find the lowest common multiple by inspection and check to see whether it is correct.

   a. 6, 4, 15
   b. 10, 15, 35
   c. 4, 5, 6
   d. 8, 3, 16
   e. 2, 3, 6, 8
   f. 7, 4, 5, 3
   g. 3, 12, 10, 15
   h. 7, 9, 16, 25
   i. 11, 4, 3, 2

Exponents

An exponent is a symbol placed at the right of and above a number. The exponent indicates how many times the number is to be taken as a factor. A negative exponent indicates how many times the reciprocal of the number is to be taken as a factor.

Any Power

Examples of positive exponents:

3^2 is another way of writing 9. The 2 written above and to the right of the 3 is called an exponent. It means that 3 is to be used twice as a factor. The number 3^2 is called a power of 3. In this case it is the second power. In the following, are given the meanings of 3^2, 6^2, and 4^2.

These are read:

3 to the second power equals 3 times 3 equals 9
6 to the second power equals 6 times 6 equals 36
4 to the second power equals 4 times 4 equals 16.

Example of a negative exponent:

$2^{-2} = (\frac{1}{2})^2 = \frac{1}{4}$

Activity (Grade 6)

a. 2^3	d. 3^3	g. 3^{-3}
b. 3^2	e. 4^3	h. 2^{-3}
c. 5^2	f. 2^5	i. 5^{-2}

Powers of Ten

Activity (Grade 6)

Do you see the pattern?

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1,000$
$10^4 = 10,000$
$10^5 = 100,000$
$10^6 = 1,000,000$

1. Using the pattern you discovered from the table, express each of these numbers as a power of ten.

   a. 10,000,000
   b. 1,000,000,000
   c. 10,000,000,000

2. Express these numbers as a number indicating powers of ten.
   Example: $2893 = 2 \times 10^3 + 8 \times 10^2 + 9 \times 10^1 + 3 \times 10^0 = 2893$

   a. 589
   b. 9000
   c. 89234
   d. 678
   e. 100
   f. 1000
   g. 10,000
   h. 100,000
   i. 1,000,000

Other Bases for Numeration

Groupings other than ten can be used as bases for a numeration system, for example, grouping in 3's, 12's, 24's.

Base-Three

The base-three uses 3 symbols namely, 0, 1, 2. Thus, 3 in the base-three system is written 10. The basic principle underlying this system is that groupings are based on three.

Activity

Group the following objects in groups of three; tell how many are left over. How would you write the base-three numeral?

No. of objects	No. of groups of three	No. of ones remaining	Symbolic representation
□ △ ○	1	0	10
○ _ □ △			
☾ △ ✕ ○ ◇			
○ ∞ △ ○ □			
7			
8			
9			

Base-Twelve

The clock, foot ruler, and calendar, which we use in everyday life, are examples of a numeration system with base-twelve.

Activity

1. Under what conditions are the following number sentences true?
   $9 + 5 = 2$ $\quad$ $6 + 4 = 10$

2. Supply the missing numbers.

   a. $3 + __ = 2$
   b. $9 + __ = 5$
   c. $__ + 11 = 6$
   d. $7 + __ = 0$

3. Make up more problems using "clock arithmetic."

Base-Twenty-Four

The method of telling time in our armed services is an example of base-twenty-four. The clock is divided into 24 equal parts. Time for each 24-hour period is shown in hours and minutes after midnight. 12 midnight is 2400, 1:00 a.m. is 0100, etc.

2:45 p.m. is 1445 = 14 hours after midnight $\quad$ 45 minutes

Activity

1. What are these times on a 24-hour clock?

   a. 1:45 a.m.
   b. 10:00 a.m.
   c. 1:30 p.m.
   d. 5:00 p.m.
   e. 11:50 p.m.

2. What are these times on a 12-hour clock?

a. 0255
b. 1050
c. 1345
d. 1820
3. 2220

3. What rule could you develop for changing 12-hour clock time to 24-hour clock time?

OPERATIONS

The four fundamental operations are addition, subtraction, multiplication, and division.

Addition

Addition is a number operation in which groups are combined.

Contests Using Key Totals

A key total is the sum of two addends which enables one to follow the pattern required to win a game.

Activity

1. Place the following numbers on the chalkboard:

1 2 3 4 5 6 7 8 9 10

Challenge the children to race the teacher in an attempt to see who gets to 50 first, each adding a number in a column in turn. Numbers can be repeated.

In the above pattern the key total is 11. That is, your number plus your opponent's number should total 11. In addition you would want to go first, and, if so, pick a 6. You would want to land on 17, 28, and 39. Let the children try to discover the pattern. Then vary it.

2. Variation

1 2 3 4 5 6 7 8 9

The key total now is 10. You will want your opponent to go first because you are not able to make a ten to start but you will be able to with your first choice regardless of what number your opponent chooses. You will want to land on 10, 20, 30, and 40.

3. Variation

1 2 3 4 5 6 7 8

Now the key total will be 9. You will want to land on 14, 23, 32, 41.

4. Variation

2 3 4 5 6 7 8

Again the key total is 10. Your opponent may go first. Numbers to land on are 10, 20, 30, and 40.

5. Variation

1 2 3 4 5 6

The key total is 7. You will want to land on 8, 15, 22, 29, 36, and 43.

6. Discovery

It would be an interesting project to see whether this statement is true - "The sum of the first and the last numbers is the key total."

Sample: 3 4 5 6

If the above statement is true, the key total should be 9. If you go first, pick a 5. Numbers you will want to land on are 5, 14, 23, 32, and 41.

7. Variation

4 5 6 7 8 9 10 11 12

The key total is 16. You will want to go first. Pick a 2. Numbers you want to land on are 2, 18, and 34.

8. Variation

2 4 6 8 10

The key total is 12. Numbers you want to land on are 2, 14, 26, and 38. You will want to go first. Pick 2.

9. Variation

1 3 5 7 9 11

Can this one work? Try it.

Commutative Principle

The Commutative Principle for addition is:

A pair of numbers can be interchanged without changing their sum.

Example: 87 plus 39 equals 39 plus 87
(Changing the order of the addends does not affect the sum.)

Activity

Work the following examples to illustrate the Commutative Principle.

a. Does 98 plus 34 equal 34 plus 98?
b. Does 108 plus 1001 equal 1001 plus 108?
c. Does 555 plus 55 equal 55 plus 555?

A Checking Procedure

One way of checking addition problems is by casting out 9's. The excess of 9's in a given number equals the excess of 9's in the sum of its digits.

Example:

```
 745     7 + 4 + 5 = 16; 1 + 6 = 7
 378     3 + 7 + 8 = 18; 1 + 8 = 9
 894     8 + 9 + 4 = 21; 2 + 1 = 3
  45     4 + 5 = 9       ---------
----                          19 = 10
2062     ---------                excess 1
         2 + 6 + 2 = 10
               excess 1
```

If the sum of a series of addends is correct, its excess of 9's will be the same as the sum of the excesses by the addends.

Activity

Add and check by casting out 9's.

a. 784 + 935 + 88 + 11
b. 935 + 811 + 735 + 34
c. 113 + 223 + 567 + 318
d. 855 + 739 + 148 + 635
e. 584 + 389 + 769 + 118

Creative Column Addition

A description follows of creative thinking in addition.

The teacher asks, 'Who likes to add long columns of figures? Let's start with a sum between 20,000 and 30,000."

A child volunteers, "26,987."

Step 1. He writes the number on the board.
Step 2. Another child offers the first addend and writes it immediately above the sum.
Step 3. The teacher supplies the second last addend so that its sum with the given addend equals 9,999.
Step 4. A child offers the third addend.
Step 5. The teacher supplies another addend so that its sum with the third addend equals 9,999.
Step 6. The teacher asks a child to supply the final addend. It should be the difference between 19,998 and the given answer.

```
 6939  5th addend (difference between answer and 19,998)
 2674  4th addend
 7325  3rd addend
 4710  2nd addend
 5289  1st given addend
-----
26,987
```

Activity

Have you discovered the pattern for finding the missing addends? Make up answers to 4-place addends and supply missing addends. Verify the results by adding columns.

Subtraction

Subtraction is a number operation in which groups are separated.

Testing a Principle

If the Commutative Principle holds for subtraction, then the difference for 83 minus 14 and 14 minus 83 would be equal; that is, 83 minus 14 equals 69 then 14 minus 83 would need to equal 69. Does it?

Activity

Work the following examples to see whether we can say that the Commutative Principle holds for subtraction.

a. Does 98 minus 34 equal 34 minus 98?
b. Does 1001 minus 108 equal 108 minus 1001?
c. Does 555 minus 55 equal 55 minus 555?

A Checking Procedure

One way of checking subtraction problems is by casting out 9's. The excess of 9's in a given number equals the excess of 9's in the sum of its digits. Therefore, the excess of 9's in the missing addend (the remainder or difference) plus the excess of 9's in the given addend (the subtrahend) equals the excess of 9's in the sum (the minuend).

Example:

4549	Excess of 9's is 4
-3683	Excess of 9's is 2
866	Excess of 9's is 2

$2 + 2 = 4$

Activity

Work these problems and check by casting out 9's.

a. 398 − 159

b. 5408 − 967

c. 18,088 − 7,599

Multiplication

Multiplication is a number operation in which equal sets (groups) are combined to obtain a product.

Relays and Arrays

Activity A (Relays)

Number of Players: Class or two groups

Materials: Scratch paper and pencil, or chalkboard and chalk

Procedure: Assign the same multiplication combination to each row. Each child multiplies the answer of the child in front of him by the multiplier given in the combination.

Example: 8 x 1

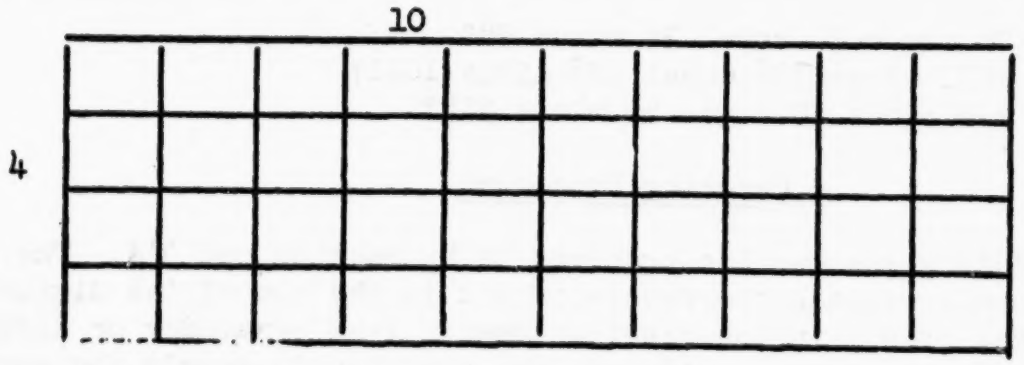
1
x8
8
x8
64
x8
512 etc.

Variation: Assign a different multiplication problem to each row.
Example: Row I, 6 x 3; Row II, 4 x 2

3	2				
x6	x4				
18	8				
x6	x4				
108	32				
x6	x4				
648 etc.	128 etc.				

Activity B (Arrays)

To illustrate the use of an array, begin by finding the products in some simple mathematical sentence, for example, n = 4 x 10. The sets are 4 and 10; the array made from squared paper would have 4 rows with 10 units in each row.



Show by arrays:

a. 3 x 12
b. 8 x 13
c. 5 x 11
d. 9 x 3
e. 8 x 14
f. 7 x 13

A Checking Procedure

One way of checking multiplication problems is by casting out 9's. Cast out 9's, then multiply the excess. This excess should equal the excess in the answer.

Example:

74	2
x48	x3
592	6
296	
3552	6

Activity

Work the problems. Then check by casting out 9's.

a. 89	c. 117	e. 112
x78	x84	x54
b. 37	d. 256	
x56	x89	

Methods of Multiplying

The methods described below give pupils a variety of ways to work multiplication problems.

Activity A (Multiplying two even numbers)

Example: 16
x14

Way to Solve: Take half of 16 which is 8; take half of 14 which is 7. Multiply 8 x 7 = 56. When you multiply $\frac{1}{2}$ by $\frac{1}{2}$ you get only $\frac{1}{4}$. Therefore, you have to multiply that product (in this problem it is 56) by 4 which is 224.

Work these problems by the above method. Check by the general method.

a. 18
x12

b. 14
x12

c. 16
x18

d. 18
x18

e. 12
x16

Activity B (Lightning method of multiplication)

Example: 68
x34

Way to Solve: Think
(30 x 60) + (30 x 8) + (4 x 60) + (4 x 8)

4 x 8 = 32 ones
32 ones = 3 tens + 2 ones
Write the 2 in the ones place.
4 x 6 tens = 24 tens
3 tens x 8 = 24 tens
24 tens + 24 tens + 3 tens = 51 tens
Write the 1 in the tens place and change 50 tens to 5 hundreds.
3 tens x 6 tens = 18 hundreds
18 hundreds + 5 hundreds = 23 hundreds
Write the 3 in the hundreds place and change 20 hundreds to 2,000. Write the 2 in the thousands place. The answer is 2,312.

Work these examples by the lightning method. Check by the general method.

a. 23 x 14
b. 13 x 12
c. 28 x 39
d. 78 x 89
e. 97 x 56

Activity C (Multiplying a fifty number by itself)

Example: 56
x56

Way to Solve: Multiply 6 x 6 = 36. Write this answer down in the first two place value positions. The left-hand figures are found by multiplying the two tens figure and adding the ones. Thus, the two left-hand figures of the product are 25 + 6 = 31. The product is 3136. Can you explain why this method works?

Work these examples by the above method. Check by the general procedure.

a. 58
x58

b. 54
x54

c. 57
x57

d. 53
x53

e. 59
x59

Activity D (Multiplying a number by itself--a two-figure number ending in 5)

Example: 35
x35

Way to Solve: Multiply 5 x 5 = 25. Write this down in the first two place value positions. The left hand figure or figures are found by multiplying the two tens figures and adding the tens figure to the product. For example, 3 x 3 = 9, 9 x 3 = 12. Write 12 in the third and fourth place value positions. The answer is 1225. Can you explain why this method works?

Work these examples by the above method. Check by using the common method.

a. 45
x45

b. 85
x85

c. 55
x55

d. 65
x65

e. 95
x95

Activity E (Doubling method)

The multiplication facts which one needs to know in order to use this method are the facts for 2.

Example: 76 x 56
For multiplying a number by this method, make a table as follows:
First write the product of 1 times the number.
1 x 56 = 56
Next, double the first product.
2 x 56 = 112
Next, double the second product. (This product is 4 times the original number.)
4 x 56 = 2 x 112 = 224
Next, double the third product. (This product is 8 times the original number.)
8 x 56 = 2 x 224 = 448

Continue this way until the table allows one to choose multipliers whose sum is the same as the multiplier of the example. Cross out the other rows.

~~1 x 56 = 56~~
~~2 x 56 = 112~~
4 x 56 = 2 x 112 = 224 224
8 x 56 = 2 x 224 = 448 448
~~16 x 56 = 2 x 448 = 896~~
~~32 x 56 = 2 x 896 = 1792~~
64 x 56 = 2 x 1792 = 3584 3584
76 x 56 = 224 + 448 + 3584 = 4256

Work these examples by the doubling method and check by another method.

a. 24 x 86
b. 62 x 259
c. 23 x 284

Activity F (Complementary Method)

To find the complement of a number, subtract it from the next power of ten. Subtract both numbers from the same power of ten. This method of multiplying is satisfactory as long as the complements of the numbers are small.

Example: 87 x 96 = 8352

100 - 96 = 4
100 - 87 = 13

87 - 4 = 83 or 96 - 13 = 83

4 x 13 = 52

Note: The first part of the answer is obtained by multiplying the difference of the factors and complements; the second part of the answer is the difference between a given factor and the difference of the other pair of complement and factor.

Try these using the above method.

a. 95 x 82

b. 87 x 93

c. 98 x 99

d. 84 x 79

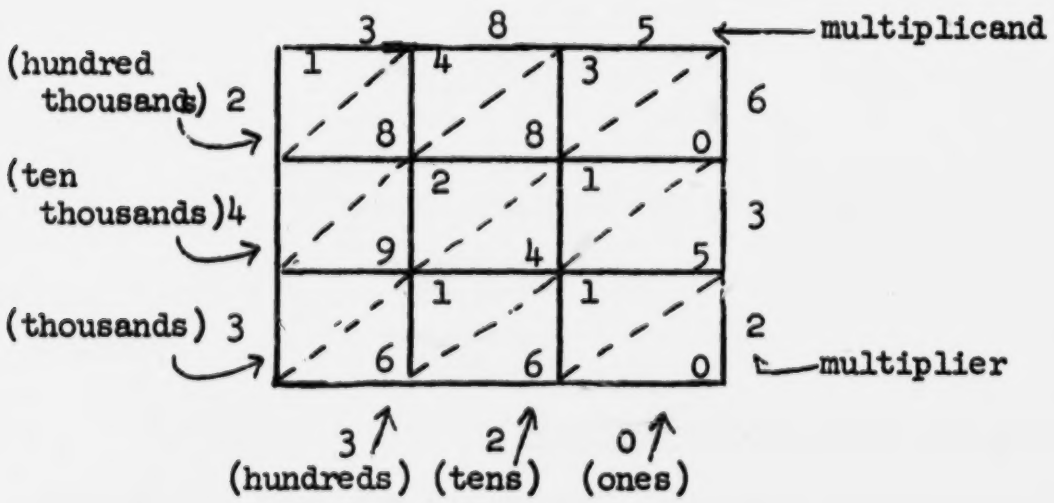
e. 96 x 75

Activity G (Lattice or grating method)

This process affords opportunity to diagram the problem.

Draw a square with 9 cells as illustrated.
Draw diagonals through each cell.

Example: 632 x 385
Write the multiplicand at the top of the square.
Write the multiplier at the right side of the square.



385
632
770
11550
231000
243,320

Begin with 2 x 5 = 10.
Write the 1 above the diagonal and the 0 below it.
Then, 2 x 8 tens = 16 tens. Write the 1 hundred above the diagonal and the 6 tens below.
Then, 2 x 3 hundreds = 6 hundreds. Write the 6 hundreds below the diagonal.
Continue the same form, multiplying each of the digits in the multiplicand first by the 3 tens and then by the 6 hundreds of the multiplier.

Start adding with the lower right-hand diagonal and carry to the next diagonal when necessary. The product appears at the ends of all diagonals used.
Check the result by multiplying the regular way.

Work these problems by the lattice or grating method.

a.	b.	c.
989	535	214
x763	x84	x328

Application of the Associative Law

Much of the work in arithmetic can be done mentally. Replace the factors by new factors to simplify the multiplication and indicate the product.

Example: $18 \times 35 = 9 \times (2 \times 35) = 9 \times 70 = 630$

Activity

Work these problems by the method used above.

a. $12 \times 45 =$	c. $24 \times 75 =$
b. $25 \times 12 =$	d. $18 \times 25 =$

Division

Division is a number operation in which a group is separated into equal groups. It is the inverse of multiplication.

Using Factors of the Divisor

Divide by using factors of the divisor and check by the long division method.

Example: $36\overline{)1872}$

What are the factors of 36?

$6 \times 6 = 36$	$4 \times 9 = 36$
$1872 \div 6 = 312$	$1872 \div 9 = 208$
$312 \div 6 = 52$	$208 \div 4 = 52$

Activity

Work these problems by using factors of the divisor. Check.

a. $56\overline{)2912}$	c. $81\overline{)5832}$
b. $27\overline{)1377}$	d. $42\overline{)3276}$

Using the Distributive Property

Divide by using the distributive property. Change the dividend to the sum of addends easily divisible by the divisor.

Example: $720 \div 3 = n$

$n = (600 \div 3) + (120 \div 3)$

$n = 200 + 40$

$n = 240$

Activity

Work the following examples by the method shown above.

a. $285 \div 5 = (200 - 5) + (80 - 5) + (5 - 5)$
b. $6100 \div 50 =$
c. $72560 \div 80 =$
d. $7272 \div 36 =$
e. $936 \div 9 =$

Laws of Divisibility

These are generalizations about divisibility. They are gradually discovered by the children.

1. A number is divisible by 2 if its units digit is 0, 2, 4, 6, or 8.
2. A number is divisible by 5 if its units digit is 0 or 5.
3. A number is divisible by 10 if its units digit is 0.
4. A number is divisible by 3 if the sum of its digits is divisible by 3.
5. A number is divisible by 9 if the sum of its digits is divisible by 9.
6. A number is divisible by 4 if its last two digits form a number divisible by 4.
7. A number is divisible by 6 if it is divisible by 2 and 3.
8. A number is divisible by 8 if its last three digits form a number divisible by 8.

Activity

1. Answer these questions.

   a. What must be added to 15 so that the new number is divisible by 2?
   b. What must be added to 124 so that the new number is divisible by 3?
   c. Can you make up a rule for divisibility by 15? What is it?

2. List numbers less than 11 that are whole number factors of a given number. Use the Laws of Divisibility for help.

Example:	Given Number	Factors
	128	2, 4, 8

a.	235	d.	1011
b.	831	e.	26,859
c.	7560	f.	128,563

More Than One Operation

Some problems require the use of more than one operation.

Estimating

The estimated answer to any problem may be obtained by rounding the number to the lower bound and upper bound and working the problem mentally.

Example:
$$\begin{array}{r} 483 \\ +247 \\ \hline \end{array}$$

Estimated Answer:	Lower bound	True answer	Upper bound
	400	483	500
	+200	+247	+300
	600	730	800

Therefore, the true answer is larger than the lower bound estimated answer but smaller than the upper bound estimated answer.

Activity

Give the estimated answer for each problem using the lower bound and upper bound. Then find the true answer.

a. $763 + 547$
b. $1987 - 256$
c 87×53
d. $23\overline{)846}$
e. $87 + 54$

Problem Solving

The following problems were supplied to the workshop members by Dr. John Marks. The children should be encouraged to use exploratory methods in solving the problems and to seek patterns in making generalizations about them.

Problem 1

What is the largest number you can write using the numerals 4 and 5 and only 4 and 5?

Problem 2

Find pairs of numbers whose product and sum are the same.

Problem 3

Given three measuring cups, with one measuring 1/2 cup, one measuring 1/3 cup, and one measuring 1/4 cup, what size measurement less than a cup can be measured?

Problem 4

Given ten tulip bulb plants, plant them so there are 5 rows and 4 bulbs in each row. Start with one row of 4 bulbs. Look for points on lines which will intersect so that the bulbs can be counted for more than one row.

Problem 5

110 tulip bulbs are planted in a series of equal rows. State all the possible arrangements of bulbs in rows.

Problem 6

Find the smallest number that can be added to 4066 so the number is divisible by 64.

Problem 7

Find the smallest number that can be added to 65,003 so the sum is divisible by 397.

Problem 8

Is it possible to have exactly 3 consecutive composite numbers between two primes? Give examples.

Problem 9

Is it possible to have 4 composite numbers between 2 primes? Can you supply examples?

Problem 10

Find 2 numbers whose product is 2 times their sum.

Problem 11

A lemon pie is cut in 7 equal parts. An apple pie is cut in 11 equal parts. Jane wants 5 pieces of lemon. Jim does not want to take more than Jane but wants as much as possible.

a. What does Jim take if he takes only apple pie?
b. What does Jim take if he takes the rest of the lemon pie first, so that he does not have more than Jane?

Problem 12

Is there a whole number K for which $3/4 = \frac{3 + K}{4 + K}$? What is it?

Problem 13

Find the sum of the first one hundred counting numbers.
Sum = 1 + 2 + 3 + ... + 99 + 100.

Problem 14

Find the sum of:
Sum = 10 + 12 + 14 + 16 + ... + 52.

Problem 15

Write 12 as the product of 2 factors so that the sum of the factors is 8; is 7.

Problem 16

Write 72 as the product of 2 factors so that the sum is 73; 17; 18.

Problem 17

At a party each child takes a piece of candy from a box and passes the box on to the next child. Joan gets the first piece, 7th piece and possibly some others. How many children are there at the party?

Draw a diagram to prove or disprove the following:

a. Could 2 children have attended the party? Why or why not?
b. Could 3 children have attended? Why or why not?
c. Could 4 children have attended?
d. Could 5 children have attended?
e. Could 6 children have attended?
f. Could more than 6 children have attended the party?

Variations

a. How many children could attend if Joan gets the first and ninth piece?
b. How many children could attend if Joan gets the first and 12th piece?

OTHER CONCEPTS

Topics treated in this section are measurement, patterns, inequalities, sets, and exploratory geometry.

Measurement

The numbers used in measurement are approximate, in contrast with counting numbers, which are exact.

Linear Measure

Linear measure is used to measure lengths and distances.

Activity

Use symbols to make each statement a true sentence.
Symbols: $<$ is less than; $>$ is greater than; $=$ is equal to.

a.	2 ft.	1 yd.	e.	25 in.	2 ft.	h.	63 in.	2 yd.
b.	15 in.	1 ft.	f.	18 ft.	6 yd.	i.	6 in.	1/2 ft.
c.	60 in.	5 ft.	g.	5 yd.	16 ft.	j.	3 ft.	40 in.
d.	37 in.	1 yd.						

Liquid and Dry Measure

In their daily living children become familiar with liquid and dry measure. However, the latter is being used less frequently.

Activity A

Use symbols to make each statement a true sentence.
Symbols: $<$ is less than; $>$ is greater than; $=$ is equal to.

a.	3 cups	1 qt.	d.	7 qt.	1 pk.	g.	2 qt.	4 pt.
b.	3 gal.	21 pt.	e.	2 bu.	8 pk.	h.	15 cups	1 gal.
c.	5 pk.	1 bu.	f.	6 pt.	1 gal.			

Activity B

Insert a number in the space provided.

a. 1 gal. is___times as large as a pt.
b. 1 cup is___as large as 1 pt.
c. 1 bu. is___times as large as 1 pk.
d. 1 qt. is___as large as 1 pk.
e. 2 qts. are___times as large as 1 pt.

Area and Volume

Children need exploratory activities to develop concepts of measuring the interior area and volume of geometric forms.

Activity A (Exploring possible dimensions of a given area)

Find dimensions of a quadrilateral whose interior area is equal to 144 sq. inches. Illustrate your solutions by diagrams using 1/2" to represent an inch. Will there be more than one set of dimensions that will satisfy the number sentence condition? About how many different solutions are there?

Activity B (Exploring possible dimensions of a given volume)

Find dimensions of a rectangular solid whose interior volume is equal to 1728 cubic inches. Will there be one set of dimensions? Why? Illustrate some solutions.

Patterns

A pattern shows a definite relationship between numbers, processes, or principles. Children should be encouraged to discover patterns.

Activity A

Examine the pattern in these sequences and supply the missing blanks. Write the rule for the pattern.

a. 1, 4, ___, 10, ___, ___, 19, ___, ___, ___.
b. 576, 288, 144, ___, ___, ___, ___, ___, $2\frac{1}{4}$, ___, ___, ___.
c. 5, 8, 10, 13, 15, ___, ___, ___, ___, ___, ___, 33, ___, ___.
d. 1, 1, 2, 3, 5, ___, ___, 21, ___, ___, ___, ___, ___.
e. 2, 7, 17, ___, ___, ___, ___, ___, ___.

Activity B (Fibonacci's progression)

Discover the sequential pattern and fill in the missing blanks.

1, 1, 2, 3, ___, ___, ___, ___, ___, 89.

Inequalities

Inequalities may be structured in a number sentence by the use of symbols.

Symbols: $<$ is smaller than; $>$ is larger than.

Activity

Put in the correct sign to make each number sentence true.

a.	$\frac{3+4}{6}$	$1 - \frac{1}{2}$	f.	54"	5'
b.	8 x 7	112 ÷ 2	g.	27 hours	2 days
c.	3/4 + 1/2	1 + 1	h.	2 gal.	4 qts.
d.	4 - 1/2	3 3/4	i.	(65 x 12)	70 x 13
e.	1010 - 9	999 + 1	j.	(900 - 873)	90 - 73

Sets

The term "sets" is replacing "groups" in the language of arithmetic.

Concept of Sets

A set is a collection of objects, of lines, of line segments, etc. Sets are enclosed in braces for notation. If the members of a set can be counted, the set is called finite. If the members of a set cannot be counted, the set is called infinite and three dots are placed before the last brace to indicate endlessness. If a set has no members, it is an empty set.

Examples:

Finite set $\{2, 4, 6, 8, 10\}$ The set denotes the even numbers less than 12.

Infinite set $\{2, 3, 5, 7 \ldots\}$ The set denotes the prime numbers.

Empty set $\{\ \}$ The set denotes all the numbers in the ones place divisible by 10.

Activity A

1. Write Set B denoting the even numbers less than 10.
   Set B =
2. Write Set C for prime numbers less than 10.
   Set C =
3. Write Set D, the set of odd numbers less than 10.
   Set D =
4. Write Set E, even prime numbers between 10 and 20.
   Set E =
5. Write Set F containing all the whole numbers which are factors of 8.
   Set F =
6. Complete this sentence:
   All these sets are _____ sets.

Activity B

1. Write Set A representing all the two-headed men.
2. Write Set B representing all the people in this room who are taller than 7 feet.
3. Complete this sentence:
   These sets are _____ sets.

Activity C

1. Write Set A representing all the counting numbers.
2. Write Set B representing all the prime numbers.
3. Write Set C representing all the composite numbers.
4. Complete this sentence:
   These sets are _____ sets.

Intersection of Sets

The intersection of sets is a set which contains only the members common to both sets.

Example:

Set A contains the members:

Set A = $\{1, 2, 3, 4\}$

Set B contains the members:

Set B = $\{2, 4, 6, 8\}$

What are the members common to Sets A and B?
Answer: Common elements are 2 and 4.

The intersection of sets is shown by the symbol $\cap$. Thus, for the above problem: $A \cap B = \{2, 4\}$

Activity

1. Write the members for Sets A and B and their intersection C equal to 5.
2. Write the members for Sets B and C and their intersection zero.
3. Write the members for Sets D and C and their intersection $\{3, 5, 7\}$.
4. Write the intersection of the set of even numbers less than 10 and the set of whole numbers from 1 to 9.
5. Write the set of whole numbers 2 through 12 and the set of whole numbers 9 through 20. Then write their intersection.

Union of Sets

The union of sets is a set which contains members of both sets without duplicating members which appear in both sets. The union of sets is shown by the symbol $\cup$.

Example: Set A = $\{2, 4, 6\}$
Set B = $\{1, 2, 3\}$

Union of Sets A and B = A $\cup$ B = $\{2, 4, 6, 1, 3\}$

Activity

1. Write the union of Sets C and D.

Set C = $\{2, 4, 8\}$
Set D = $\{2, 3, 6\}$
C $\cup$ D = $\{\quad\}$

2. Make up 5 more problems with 2 sets each and show their union.

Exploratory Geometry

From their environment, pupils get intuitive ideas about geometric forms. With teacher-guidance, pupils may be led to explore and find properties of geometric figures.

Points and Lines

A point is an idea or concept; what we see is a picture of the idea or concept. Capital letters are used to name or label points. A line is a set of points and is unending. The symbol for line AB in the example below is $\overleftrightarrow{AB}$.

Examples: Points

A. .B

Line

Activity

1. Given two points, A and B:

.B

A.

Draw a straight line passing through points A and B. Extend the line beyond A and B. Can you draw another line through A and B?

2. Given three points not on the same line, how many lines can you draw? Illustrate.

3. Given four points, you are to join them a pair at a time. How many lines can you draw? Illustrate.

4. Take 4 points again. This time try another arrangement. The fourth point is at the tip of your finger. How many lines can you draw, joining only one pair at a time?

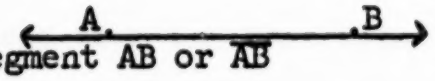
5. Tabulate your discoveries to complete the following table:

No. of Points	2	3			
No. of Lines	1				

Look for a pattern.
Can you find a rule?
(2 x 3 = 6 ÷ 2 = 3) Hence, 3 points, 3 lines.
(3 x 4 = 12 ÷ 2 = 6) So, 4 points, 6 lines.

Lines and Line Segments

A line is a set of points; it is unending. A line segment is a portion of a line with two end points. The symbol for line segment AB in the example below is $\overline{AB}$.

Example: 

Line segment AB or $\overline{AB}$

Lines and line segments may or may not intersect; the intersection is the member or members common to both. The union of lines and line segments can be found.

Interpretation of symbols:

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD}$ means that lines AB and CD intersect.
$\overline{AL} \cup \overline{CD}$ means the union of line segments AB and CD.

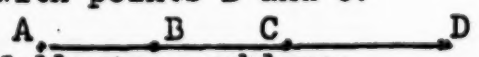
Activity A

1. Draw $\overline{AB}$ and $\overline{CD}$ so that $\overline{AB} \cap \overline{CD}$ is empty.
2. Draw $\overline{AB}$ so that line $AB \cap \overline{CD}$ is 1 point.

3. How many points are on line BA?
4. How many points are on line CD?

Activity B

Given $\overline{AD}$ with points B and C:



Work the following problems:

a. $\overline{AB} \cap \overline{BC}$
b. $\overline{AB} \cap \overline{CD}$
c. $\overline{AC} \cup \overline{BC}$
d. $\overline{AD} \cup \overline{BC}$
e. $\overleftrightarrow{AB} \cap \overleftrightarrow{AB}$
f. $\overleftrightarrow{AB} \cup \overleftrightarrow{CD}$

Activity C

Given:

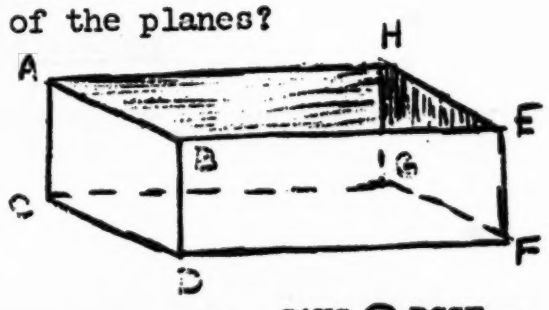
1. Write or name 2 segments whose intersection is a segment.
2. Name 2 segments whose intersection is a point.
3. Name 2 segments whose intersection is empty.
4. Draw $\overleftrightarrow{AB} \cap \overleftrightarrow{CD}$ as a point.

Planes

A plane is a flat surface. It is unlimited in extent. Any three points not on the same line are in one and only one plane.

Activity

1. Examine a shoe box. Label the points of the shoe box.
2. Name the planes of the shoe box.
3. How many different planes make the box.
4. What are the intersections of the planes?



5. Work these problems.
   a. Plane ABEH $\cap$ DBEF =
   b. Plane ABDC $\cap$ DCGF =
   c. CAHG $\cap$ DCGF =
   d. HEFG $\cap$ DCGF =

EXTENSION OF THE NUMERATION SYSTEM

Since our numeration system of whole numbers was inadequate to meet the needs of society, man extended the system by inventions including fractions, decimals, and per cent. To do this man reordered the symbols of numbers.

Fractions

A fraction is an ordered pair of numbers.

Concepts

Activity A

Supply the missing number in each box.

Given fraction	1/6	1/5	1/4	1/3	1/2	1/8
Write a fraction 3 times as large as the fractions in the first row.						
Write a fraction twice as large as the given fraction.						
Write a fraction half as large as the given fraction.						
Write a fraction that tells how much to add to the given fraction to get a sum equal to 1.						
Write a fraction whose denominator is one more than the given fraction.						

Activity B

Use different names for the same number. Make a Fraction Table.

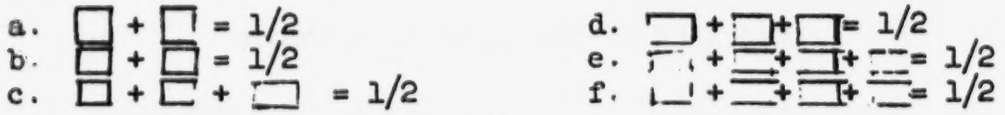
	When cut into 2 parts	When cut into 3 parts	When cut into 4 parts	When cut into 5 parts	When cut into 6 parts
1/2 =	2/4	3/6	4/8	5/10	6/12
1/3 =	2/6	3/9	4/12	5/15	6/18
1/4 =		3/12			
1/5 =			4/20		
1/6 =				5/30	

Operations

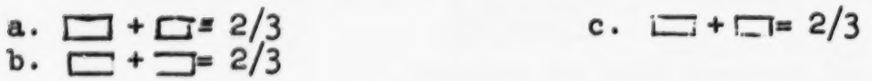
The fundamental operations of whole numbers are applicable to operations with fractions.

Activity A

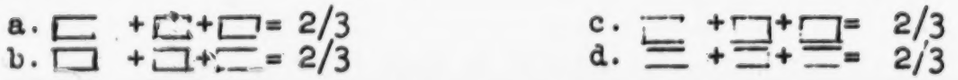
1. Given the answer 1/2, make up a number sentence by supplying numbers in the frames. The frames must contain a unit fraction.



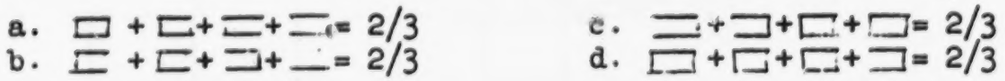
2. Given the answer 2/3, supply unit fractions in the number frames to make a true number sentence.



3. Write 2/3 as the sum of 3 unit fractions.



4. Write 2/3 as the sum of 4 unit fractions.



Activity B

The most frequent use in arithmetic for the lowest common multiple (L.C.M.), commonly known as the least common denominator (L.C.D.), is in the addition and subtraction of fractions of unlike denominators.

Example:

Find the L.C.D. or L.C.M. of 4/8, 2/6.

Prime factors of 8 = 2, 2, 2

Prime factors of 6 = 2, 3

L.C.M. = $2 \times 2 \times 2 \times 3 = 2^4$

Find the L.C.D. or L.C.M. of:

a. 7/12, 3/8	e. 4/9, 8/15	i. 5/10, 7/8, 3/6
b. 2/6, 4/15	f. 2/30, 3/4, 5/6	j. 4/16, 3/4, 9/36
c. 5/21, 5/6	g. 3/4, 2/20, 7/14	k. 4/15, 2/9, 5/6
d. 7/10, 4/6	h. 4/10, 5/12, 3/4	l. 1/2, 2/3, 7/10

Activity C

The greatest common factor (G.C.F.) is useful in the reduction of fractions.

Example: 30/48
Prime factors of 30 are 2, 3, 5.
Prime factors of 48 are 2, 2, 2, 2, 3.
Factors common to 30 and 48 are 2 and 3.
G.C.F. of 30 and 48 is 2 x 3 = 6.
30/48 = 5/8

Find the largest number that will divide both terms of each of the following fractions and change the fraction to its lowest terms.

a. 24/60	c. 21/24	e. 30/142	g. 77/88
b. 12/16	d. 20/45	f. 90/100	h. 20/24

Activity D

Fill in the blanks with the proper symbol:
$>$ means "is larger than"
$<$ means "is less than"
$=$ means "is equal to."

a. 3/7 + 4/7 _____ 11/14 + 2/14	d. 2/4 + 2/4 _____ 1/4 + 1/4
b. 1/2 + 1/3 _____ 1/6 + 5/6	e. 1/3 + 2/3 _____ 1/6 + 3/6
c. 3/5 + 1/5 _____ 2/5 + 1/5	f. 2/8 + 1/4 _____ 3/4 + 1/4

Activity E

1. Write true or false in front of each of these number statements.

a. ____ 5/8 = 1/4 + 3/8	f. ____ 13/10 = 2/5 + 7/10
b. ____ 5/6 = 1/6 + 1/2	g. ____ 15/12 = 7/12 + 2/3
c. ____ 6/10 = 2/5 + 1/10	h. ____ 23/12 = 5/6 + 2/3 + 5/12
d. ____ 7/12 = 5/12 + 1/6	i. ____ 13/8 = 1/2 + 7/8 + 1/4
e. ____ 4/12 = 1/4 + 1/12	j. ____ 19/12 = 3/4 + 1/2 + 15/12

2. Write a true number sentence for the ones that were false.

Activity F

This activity uses the "golden rule" of fractions which is: multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction, only renames it.

Example:
When we multiply both numerator and denominator of the fraction 3/4 by the same factor (not zero), we always obtain a fraction naming the same number as 3/4 names. If the factor is 3:

$\frac{3 \times 3}{3 \times 4} = \frac{9}{12}$, we obtain 9/12; if it is 2, we obtain 6/8.

Write four other fractions naming each of the following numbers:

a	2/3	___	___	___	___
b.	2/5	___	___	___	___
c.	7/8	___	___	___	___
d	1/6	___	___	___	___
e.	1/3	___	___	___	___
f.	3/5	___	___	___	___

Activity G (Grades 5-6)

Work these exercises to test understanding of concepts. Complete each statement.

a. If 2/3 + 2/3 = 1 1/3 then $\square$ x 2/3 = 1 1/3
b. If 1/2 of 1 1/2 = 3/4 then 3/4 + n = 1 1/2
c. If 4 x 3/8 = 1 1/2 then 3/8 + 3/8 + n + n = 1 1/2
d. If 1/6 of 2 2/5 = 2/5 then 2/5 + 2/5 + 2/5 + ? + ? + ? = 2 2/5

Activity H (Grades 5-6)

Work these exercises to test understanding of exponents. Write T or F to indicate if a statement is true or false.

a ___ $(7/2)^4$ = 7/2 x 7/2 x 2/7 x 7/2
b. ___ $(1\ 1/3)^3$ = 1 1/3 x 1 1/3 x 1 1/2
c. ___ $(1/4)^3$ = 1/64
d. ___ $(3/5)^2$ = 9/25

Activity I (Grades 5-6)

This activity tests the commutative property of fractions.
Supply the symbol =, >, or < in the blanks to make the number sentences true.

a. 4/7 x 5/9 $\square$ 5/9 x 4/7
b. 17/47 x 11/23 $\square$ 11/23 x 17/47
c. 8/21 x 35/96 $\square$ 11/13 x 65/100
d. 84/57 x 38/91 $\square$ 38/91 x 84/57
e. 57/52 x 65/87 $\square$ 36/55 x 33/84

Activity J (Grades 5-6)

This activity tests the associative property of fractions.
Answer the questions.

a. Does (2/3 x 5/7) x 28/15 = 2/3 x (5/7 x 28/15)?
b. Does 4/11 x (7/8 x 5/6) = (4/11 x 7/8)x 5/6?
c. Does (11/14 x 2/3) x 18/23 = 4/15 x (12/17 x 9/11)?

Activity K (Grades 5-6)

This activity tests the distributive property of fractions.
Solve each exercise in two ways.

a. 4/5 (2/3 + 3/7)
b. 7/8 (3/14 + 2/21)
c. 6/13 (7/18 + 5/24)

Activity L (Cross product method - Grades 5-6)

Use the cross product method to determine which of two fractions is the larger.

Which is larger?

Example A
3/8 or 5/8
3/8 ? 5/8
3/8 < 5/8

Example B
5/12 or 2/5
5/12 ? 2/5
?

Observe the pattern:

3 x 8 < 5 x 8
∴ 3/8 < 5/8

5 x 5 > 2 x 12
∴ 5/12 > 2/5

How can we arrive at the answer without changing to common denominators? Through analysis of the pattern we note that the first fraction is larger if the product of the first and 4th terms is larger than the product of the 2nd and third terms.

Work these problems by the above method.

Which is larger?

a. 5/6 or 7/8?
b. 3/4 or 2/3?
c. 5/12 or 6/11?
d. 3/8 or 2/5?
e. 11/16 or 14/15?

Activity M (Grades 5-6)

In this activity the dividend and the divisor are expressed as fractions having a common denominator. Direct division is possible.

Example: 6 ÷ 2/3

Change the whole number 6 to a common fraction with the denominator being 3. (18/3 ÷ 2/3). Divide 18 by 2. The result is 9.

Work these problems by the common denominator method.

a. 5 ÷ 3/4 =
b. 3 ÷ 5/6 =
d. 8 ÷ 2/3 =
d. 1/2 ÷ 2/3 =
e. 7/15 ÷ 3/4 =

Activity N (Grades 5-6)

In this activity multiply the divisor and the dividend by the Lowest Common Denominator (Least Common Multiple) to get rid of the denominator.

Example: 2/3 ÷ 3/4
L.C.D. = 12
(12 x 2/3) ÷ 12 x 3/4 8 ÷ 9 = 8/9

Work the following examples by the above method.

a. 2/5 ÷ 1/4 = n
b. 5/8 ÷ 2/5 = n
c. 1/2 ÷ 2/3 = n
d. 5/7 ÷ 1/3 = n
e. 2/5 ÷ 1/2 = n
f. 7/8 ÷ 2/3 = n
g. 2/5 ÷ 1/6 = n
h. 5/12 ÷ 2/5 = n
i. 5/9 ÷ 3/5 = n

Activity O (Grade 5-6)

In each of the examples below multiply the dividend and divisor by the same number so that the divisor is a whole number before dividing.

Example:
$4 \div 3/4 = (4 \times 4) \div (4 \times 3/4) = 16 \div 3 = 5\ 1/3$
Work these problems by the above method.

a. $8 \div 2/5 = n$
b. $20 \div 4/5 = n$
c. $12 \div 2/3 = n$
d. $4 \div 8/12 = n$
e. $16 \div 4/3 = n$
f. $14 \div 2/5 = n$
g. $24 \div 4/7 = n$
h. $16 \div 2/9 = n$
i. $18 \div 2/3 = n$

Activity P (Grades 5-6)

This exercise develops readiness for the division of fractions using the "inverted divisor" method.

Example:
Write the reciprocal of 3/5.
The reciprocal of 3/5 is a number such that when 3/5 is multiplied by it the product is 1. Thus, the reciprocal of 3/5 is 5/3 because $3/5 \times 5/3 = 1$.

Write the reciprocals of the following fractions:

a. 1/3
b. 1/2
c. 1/4
d. 1/5
e. 1/6
f. 1/9
g. 1/12
h. 1/25
i. 3/8
j. 7/9
k. 6/7
l. 11/12
m. 8/15
n. 6/13
o. 7/5
p. 9/8
q. 7/2
r. 5/3
s. 6/5
t. 3/2

Activity Q (Grades 5-6)

Every division can be written as a fraction; so, the division of one fraction by another can be written as a fraction.

Example:
$$3/5 \div 2/7 = \frac{3/5}{2/7}$$
This kind of fraction is called a complex fraction.

Step 1. Write the example as a complex fraction.
Step 2. Multiply both the numerator and the denominator by the same number to obtain a product of 1 in the denominator. Use as that number 7/2, the reciprocal of the denominator.
$$\frac{3/5}{2/7} = \frac{3/5 \times 7/2}{2/7 \times 7/2} = \frac{3/5 \times 7/2}{1}$$
Step 3. A fraction with 1 as denominator is equal to the numerator,
$$\frac{3/5 \times 7/2}{1} = 3/5 \times 7/2$$
Step 4. The division example has been changed now to a multiplication example. So in a division example, if you invert the divisor and multiply, you are merely leaving out steps 1, 2, and 3.

Work these examples by inverting the divisor.

a. $4/5 \div 2/3$	c. $3/4 \div 3/8$	e. $8 \div 2/5$
b. $7/8 \div 3/4$	d. $1/4 \div 2/3$	f. $9 \div 3/4$

Activity R (Grades 5-6)

This is another method of dividing fractions, using the reciprocal of the divisor. Eliminate inverting the divisor; multiply the dividend and the divisor by the reciprocal of the divisor.

Example:

$3/4 \div 4/5 = (3/4 \times 5/4) \div (4/5 \times 5/4) = 3/4 \times 5/4 = 15/16$

a. $2/5 \div 3/7$	e. $2/7 \div 5/6$	i. $5/9 \div 2/3$
b. $5/6 \div 2/3$	f. $3/4 \div 3/2$	j. $5/9 \div 5/6$
c. $9/10 \div 8/9$	g. $4/11 \div 5/12$	k. $7/12 \div 4/3$
d. $2/3 \div 5/7$	h. $3/10 \div 6/13$	l. $3/13 \div 1/2$

Activity S (Grades 5-6)

These exercises use more than one fundamental operation.

1. Fill in the blanks.

a. ____ x ____ = 1/8
b. (____ + ____) x ____ = 1/8
c. (____ + ____) x (____ + ____) = 1/8
d. ____ x ____ = 1/6
e. (____ + ____) x ____ = 1/6
f. ____ x ____ = 5/8
g. ____ x ____ x ____ = 5/8
h. ____ ÷ ____ = 3
i. ____ ÷ ____ = 1 1/2
j. ____ ÷ ____ = 18

2. Supply the missing symbols to make the number sentences true.

a. $5 \times 1/2 \square 5$	f. $4\ 3/4 \square 2 \times 2\ 3/8$
b. $3 \div 2/3 \square 3$	g. $8 \times 32 \square 250$
c. $9 \div 3/4 \square 12$	h. $n < 3\ 1/2$, $n = \square$
d. $5 \times 3/4 \square 3/4 \times 20/4$	i. $5 \times n = 4\ 1/6$, $n = \square$
e. $8\ 1/3 \div 7\ 1/2 \square 1$	j. $1/2 \div 1/4 = \square$

Activity T (Grades 5-6)

Use the ratio idea to find N.

Example:

To find N in 2/3 = N/12, think

4 x 3 = 12
4 x 2 = N
N = 8

Example:

To find N in 15/18 = 5/N, think

15 ÷ 3 = 5, so
18 ÷ 3 = N
N = 6

Work these problems

a. 5/6 = N/12
b. 3/4 = N/8
c. 3/8 = N/24
d. 15/24 = 5/N
e. 12/16 = 3/N
f. 48/96 = 1/N
g. 35/75 = 7/N
h. 18/54 = N/3

Activity U (Grades 5-6)

Supply the missing symbols to make the number sentences true. Use $>$, $<$, or $=$.

a. 3 x 2/3 ☐ 3
b. 3/4 x 8 ☐ 8 x 6/8
c. 1/8 x (5/6 x 3/4) ☐ (1/8 x 5/6) x 3/4
d. 1/2 of 5/6 ☐ 3/4 of 5/6
e. 3/4 + 3/4 + 3/4 + 3/4 ☐ 4 x 3/4
f. 3/8 of 24 ☐ 1/2 of 24
g. 5/3 + 5/4 ☐ 5 (1/3 + 1/4)
h. 5 1/4 ÷ 3/4 ☐ 5 1/4
i. 1/2 ÷ 1/4 ☐ 1/2
j. 3/4 x 3/8 x 3/5 ☐ 3 (1/4 x 1/8 x 1/5)

Patterns Using Fractions

Magic squares and sequences can illustrate patterns using fractions.

This is the procedure to use in making a magic square consisting of 16 cells. Think of 16 consecutive fractional numbers. Place these numbers in order of their size, in sequential order in the cells of the square. Interchange the numbers in the corner and middle cells. Prove that the resulting square is a magic square. To be a magic square all the sums of the rows, diagonals, and columns must be equal in value.

Example:

Before interchanging fractions

1/4	1/2	3/4	1
1 1/4	1 1/2	1 3/4	2
2 1/4	2 1/2	2 3/4	3
3 1/4	3 1/2	3 3/4	4

After interchanging fractions

4	2/4	3/4	3 1/4
1 1/4	2 3/4	2 1/2	2
2 1/4	1 3/4	6/4	3
1	3 1/2	3 3/4	1/4

Activity A (Grades 5-6)

Multiply each of the numbers in the magic square above by 5. Place each product in the corresponding cell of the square below. Is the resulting square a magic square? Prove your answer.

Activity B (Grades 5-6)

This is an exploratory exercise. Supply the missing numbers to make magic squares.

a.

2 1/4		1 3/4
	1 1/2	
1 1/4		3/4

b.

	1/2	
1		2
1 1/4	2 1/2	

c.

	5	
4		2
	1	

d.

2		1 1/2
	1 1/4	
1		1/2

e.

	1/4	
3/4		1 3/4
	2 1/4	

f.

2		3
	3 1/2	
4		5

Activity C (Grades 5-6)

1. Construct a magic square having 9 cells. Using fractions in some sequential order, place them according to the pattern indicated by the ordinal numbers in the square below.

Starting Cell ↓

8th	1st	6th
3rd	5th	7th
4th	9th	2nd

2. Test to prove that you have made a magic square.

Activity D (Grades 5-6)

This activity deals with fractional sequences.

1. Discover the pattern. Write 5 more terms for each.
   a. 3/8 - 1 1/8 - 1 7/8 - ____ ____ ____ ____ ____
   b. 1 1/3 - 2 2/3 - 5 1/3 - 10 2/3 - ____ ____ ____ ____ ____
   c. 9 - 8 7/8 = 9 1/8 - 9 - 9 1/4 - ____ ____ ____ ____ ____

2. Study each sequential pattern. Fill in the missing term.
   a. 1/4 - 3/4 - 6/4 - 18/4 - 36/4 - ____ ____ ____
   b. 9/2 - 18 - 36 - ____ ____ ____ ____
   c. 1/8 - 1/4 - 3/8 - 1/2 - ____ ____ ____ ____
   d. 3/4 - 6/8 - 9/12 - ____ - ____ ____
   e. 5/8 - 15/8 - 15/16 - 45/16 - 45/32 - ____ ____

3. Discover the pattern and fill in the ∠.

a.	10 2/3	10 1/3		9 2/3		9				
b.	4 3/4	4 6/8	4	4	32	4				
c.	3 3/4	3 1/4		2 1/4		1 1/4		1/4		
d.	6 1/4	6 2/8	6	16	6 8/32	6				
e.	8 1/2	8 1/4	∠	8 1/16						

Problem Solving

In problem solving, work and solve for N. Follow these steps:

1. Write the number sentence.
2. Write the word that tells which operation must be used.
3. Then solve the problem.
4. Interpret the result.

Problem 1 will serve as an example.

Problem 1

Mr. Smith gave his son John, a camera and 5 rolls of film for his birthday. John was told not to use all of his film in one day. He used 1/2 of a roll on Monday, 1 1/4 rolls on Tuesday, and 2 1/8 rolls on Wednesday. Within the three days how much film did John use? How much film did he have left?

Part 1

Number Sentence: 1/2 + 1 1/4 + 2 1/8 = N
Operation: addition
Solution:
$$\begin{aligned} 1/2 &= 4/8 \\ 1\ 1/4 &= 1\ 2/8 \\ 2\ 1/8 &= 2\ 1/8 \\ \hline & 3\ 7/8 \end{aligned}$$
Interpretation: He used 3 7/8 rolls of film.

Part 2

Number Sentence: N + 3 7/8 = 5
Operation: subtraction
Solution:
$$\begin{aligned} 5 - 3\ 7/8 &= N \\ 5 &= 4\ 8/8 \\ -3\ 7/8 &= 3\ 7/8 \\ \hline & 1\ 1/8 \end{aligned}$$
Interpretation: He has 1 1/8 rolls of film left.

Problem 2

Jim's troop made Indian headdresses. They cut leather into strips and sewed the ends of the strips together. Each boy received four strips of leather. The strips measured as follows: 1/2 yd., 7/8 yd., 2/3 yd., and 1/6 yd. How long a piece would this be when the strips were sewn together?

Problem 3

Jane was making a skirt with 4 rows of ruffles. The first row was 1/2 ft. long; the second row was 1 1/4 ft. long; and the third row was 1/3 ft. long. Jane wanted the skirt to be 3 ft. long. How long should the fourth row of ruffles be?

Problem 4

Mr. Jones painted his garage. He bought 4 gals. of paint. When he was finished painting he had 2/3 gal. left. How much paint did he use?

Problem 5

Mrs. Green wishes to make two dresses for her daughter, each of which will require 2 3/4 yds. of material. Mrs. Green has 4 1/2 yds. of material. How much more material must she buy?

Problem 6

Three years ago, I weighed 65 pounds. Today my weight is 1 3/4 times as much as it was three years ago. What is my present weight?

Problem 7

How many pieces of rope, each 3 2/3 ft., can be cut from a piece 51 1/3 ft. long?

Problem 8

Jim's weight was 125 1/2 lbs. before he went to camp. Now upon his return from camp, he weighs 138 1/4 lbs. How many pounds did he gain?

Problem 9

Joe is going to ride on a train for 2 1/2 hrs. The train travels 50 3/4 miles per hour. How far will Jim travel?

Problem 10

A Scout Troop were washing cars in order to add to their treasury. John washed a car in 3 2/3 hrs. Tom did a car in 2 3/4 hrs. How much longer did John take to do his work?

Problem 11

There are 710 students in our school of which 2/5 are boys. How many girls attend our school?

Problem 12

In the sporting goods store there is a catcher's glove that sells for $8.40. Bob said he would pay 3/8 of the cost, David would pay 1/3, and Rickey the rest. What fractional part did Rickey pay? What amount of money did Rickey pay? Bob? David?

Problem 13

Mr. Jones receives $20.00 a day for working eight hours. Today he received $12.50. What fractional part of the day did he work?

Problem 14

A bottle of salad dressing was 3/5 full last week. Now it is 1/3 full. How much was used?

Problem 15

Which is greater and how much greater: 5/6 of a yard or 28 3/4 inches?

Decimals

Decimal numerals are an extension of our system of numeration. The units place is the center of our numeration system. Any digit to the left of the units place occupies a position ten times as great; any digit to the right of the units place occupies a position one-tenth as large as the units place.

Place Value Chart

A system of exponents can be used to show decimal place value. (See chart below.)

Place Value Chart
Fractions, Exponents, and Decimals

1,000,000	100,000	10,000	1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$(10)^6$	$(10)^5$	$(10)^4$	$(10)^3$	$(10)^2$	$(10)^1$	$(10)^0$	$(10)^{-1}$	$(10)^{-2}$	$(10)^{-3}$
1,000,000	100,000	10,000	1,000	100	10	1	.1	.01	.001
Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Tens	Ones	Tenths	Hundredths	Thousandths

Activity (Grade 6)

Using the above chart, compute the following:

a. $6\ (10)^3 + 7\ (10)^2 + 5\ (10)^1 + 2\ (10)^0 = 6752$
b. $8\ (10)^2 + 4\ (10)^1 + 3\ (10)^{-1} =$
c. $4\ (10)^{-1} + 6\ (10)^{-2} =$
d. $(10)^{-1} + 0\ (10)^{-2} + 8\ (10)^{-3} =$
e. $7\ (10)^{-1} + 0\ (10)^{-2} + 8\ (10)^{-3} =$
f. $2\ (10)^4 + 20\ (10)^{-3} =$
g. $1\ (10)^6 _ 1(10)^0 + 1\ (10)^{-1} =$
h. $3\ (10)^3 + 3\ (10)^{-3} =$

Estimation

An estimate is an approximate answer to a problem. We generally estimate by rounding numbers and using mental arithmetic. This procedure enables us to compare our exact answer with a logical guess and to extend principles and generalizations to new numbers invented by man.

Activity (Grade 6)

Estimate the products of factors involving decimals by the use of lower and upper bound.

Example: Estimate the answer to 2 x 2.7.
Solution:
Between what two whole numbers could one locate 2.7 on the number line? (2 and 3)
What is the lower bound or lower limit of 2.7? (2)
What is the upper bound of 2.7? (3)

Lower bound	Exact answer	Upper bound
2	2.7	3
x2	x 2	x2
4	5.4	6

Therefore, the product of 2 and 2.7 is greater than 4, the lower bound, and smaller than 6, the upper bound.

Estimate the answers to these problems giving the lower and upper bounds.

a. 7 x 3.6
b. 1.7 x 54
c. 3.5 x 13.5
d. 6 x 4.2
e. 7 x 3.9
f. 5 x 4.16
g. 8 x 3.7
h. 5 x 12.8
i. .62 x 35
j. 4.6 x 40.9

Multiplication of Decimals

Activity A (Grade 6)

Complete the following table.

Exercise	Product Expressed As	
	Common Fraction	Decimal Numeral
a. 2/10 x 3/10		
b. 2/10 x 54/100		
c. 4/10 x 5/1000		
d. 4 x 6/10		
e. 6 x 2/100		
f. 3 x 3/1000		
g. 4/100 x 6/10		
h. 2/100 x 6/100		
i. 4/100 x 2/1000		

Activity B (Grade 6)

From the preceding chart, make generalizations to complete the statements listed below.

a. Number of tenths x number of tenths = number of hundredths.
b. Number of tenths x number of hundredths = number of _____.
c. Number of tenths x number of thousandths = number of _____.
d. Number of hundredths x number of hundredths = number of _____.
e. Number of ones x number of tenths = number which contains _____.
f. Number of ones x number of hundredths = number which contains _____.
g. Number of ones x number of thousandths = number which contains _____.

Activity C (Grade 6)

1. Fill in the blanks in the table below.

I Exercise	II No. of decimal places in multiplier	III No. of decimal places in multiplicand	IV Product	V No. of decimal places in product
a. 2 x .3	0	1	.6	1
b. .3 x .4				
c. .12 x .3				
d. 23.2 x .03				
e. .002 x .18				
f. 12 x .03				
g. 4 x .007				
h. .5 x .025				
i. .4 x .04				
j. .5 x .5				

2. Answer this question. The number in column V is equal to the numbers in which columns?

3. From the findings in this table state a rule about placing the decimal point in the product.

Activity D (Grade 6)

Use the place value method (or grid method) for multiplying decimal fractions. Write the numerals for the multiplicand and the multiplier in their proper decimal value positions. Find the partial products by multiplying the digits or numbers in the multiplicand by the integer that is the multiplier. Then add the partial product to find the total product or the number whose value equals all the decimal fractions combined.

Example: .28
x 36.

.48	(6 ones x 8 hundredths)
1.2	(6 ones x 2 tenths)
2.40	(3 tens x 8 hundredths)
6.0	(3 tens x 2 tenths)
10.08	

Compute the following problems by the method shown above:

a. 6.2 x .8 b. .42 x2.7 c. 6.03 x68.5 d. 22.2 x20

Activity E (Grade 6)

Find n; then write it as a decimal numeral.

Common fraction	Decimal fraction
a. 3/4 x 1/2 = n	.75 x .5 = .375
b. 1/2 x 2/5 = n	
c. 1/4 x 3/5 = n	
d. 1/2 x 4/5 = n	
e. 3/8 x 1/2 = n	

Per Cent

A per cent represents a relationship in which a number is compared to 100. This relationship is known as a ratio.

It has been a common practice to use per cent incorrectly or to misuse and abuse quantitative terms involving per cent. Too little emphasis has been placed on the sound arithmetic interpretation of per cent.

Activity A (Grade 6)

1. Read each statement carefully. Place the letter C before the statements that use per cent correctly. Place the letter I before the statements that use per cent incorrectly. Place the letter M before the statements that may be correct but are misleading.

   a. ( ) I feel 100% better today than I did yesterday.
   b. ( ) Our school enrollment increased from 200 to 600 in 5 years. This is an increase of 300%.
   c. ( ) My batting average is 300 per cent.
   d. ( ) I am 15% heavier than Sue.
   e. ( ) Our detergent gets clothes 30% brighter.
   f. ( ) Miami is 25% warmer than Denver.
   g. ( ) The water content is 75%.
   h. ( ) This grass seed is guaranteed to contain less than 5% weeds.
   i. ( ) Prices are reduced 100%.
   j. ( ) Prices reduced from $10.00 to $5.00 have a 100% reduction.
   k. ( ) The Gloom Tobacco Co. claims that 60% of the students at State U. prefer Gloom King Size.
   l. ( ) The Smallsville Gazette boasted that 50% of Smallsville's students at State U. made the dean's list.
   m. ( ) 50% of all the students at City High School will go to State U. This means that, in our room, 16 out of 32 students will go to State U.

2. At the end of the incorrect and misleading statements above, place the letter corresponding to the reason the statement is incorrect or misleading. Letters and reasons are listed below.

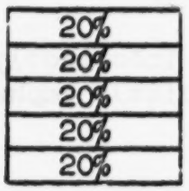
   Reasons for the statements being incorrect or misleading:
   a. Compares things that cannot be measured
   b. Compares amounts other than those implied in the problem (Answer given is incorrect.)
   c. Object to which comparison is made is not given or is not clear
   d. Not significant unless the whole group is large
   e. Comparison not necessarily valid for a part of the sample
   f. Mathematically possible but very unlikely to happen
   g. Comparison is made to a number other than 100.

Activity B (Grade 6)

Illustrating the problem by a diagram often helps the child to organize his thinking.

Problem:
20% of the cost of a trip was $60. What is the cost of the trip?

1. Draw a sketch by marking off a rectangular shape to represent 20%. Continue adding more rectangles until the sum of the per cents equals 100%.



2. Answer these questions:
   How much money is represented by 20%?
   How many $60's do we have indicated on the diagram?
   What was the cost of the trip?

3. Make up another problem and a diagram to support the answer.

Activity C (Grade 6)

Use the intuitive method in working with per cent.

1. Study Example A:

   15% of 40 = n
   Find 10% of 40; .10 x 40 = 4.
   Find 5% of 40; 5% of 40 = 1/2 of 4 = 2
   15% of 40 = 6

2. Work these exercises by the intuitive method.
   a. 12 1/2% of 40 = n
   b. 15% of 50 = n
   c. 35% of 40 = n
   d. 45% of 40 = n
   e. 95% of 50 = n

3. Study Example B:

   20% of _____ = 16
   Find 100% of number = 5 x 20% = 5 x 16 = 80

4. Work these exercises by the intuitive method.

   a. 15% of _____ = 18
   b. 5% of _____ = ?
   c. 100% of _____ =
   d. 175% _____ = 56
      25% _____ =
      100% _____ =
   e. 1/2% of _____ = 3
      1% of _____ =
      100% _____ =

5. Study Example C:

   _____% of 40 = 6
   10% of 40 = 4
   5% of 40 = 2
   15% of 40 = 6

6. Work these exercises by the intuitive method.
   a. _____% of 40 = 8
   b. _____% of 50 = 7 1/2
   c. _____% of 40 = 18
   d. _____% of 40 = 5
   e. _____% of 50 = 47 1/2

Activity D (Grade 6)

1. Work these problems mentally.
   a. Find mentally 37 1/2% of 64. Change 37 1/2% to a fraction, then work the problem.
   b. Find 62 1/2 % of 48.
   c. Find 16 2/3% of 33.
   d. Find 75% of 32.
   e. Find 80% of 40.

2. Work these problems mentally.
   a. What per cent is 18 of 30?
      First find what fractional part one number is of the other, then change the fraction to a per cent.
   b. What per cent is 2 of 3?
   c. What per cent is 16 of 64?
   d. What per cent is 24 of 96?
   e. What per cent is 15 of 90?

3. Work these problems mentally.
   a. 5 is 20% of what number?
      What is 1% of the number?
      What is 100% of the number?
   b. 5 is 50% of what number?
   c. 28 is 7% of what number?
   d. 25 is 5% of what number?
   e. 18 is 9% of what number?

Activity E

Use the equal ratio method in working these examples.

1. Study these problems.

Example A:
Candy bars sell at 3 for 10¢. What is the cost of 6 candy bars?

$$3/10 = 6/n$$

$$\frac{3 \times 2}{10 \times 2} = 6/n \qquad n = 20$$

Example B:

$$20\% \text{ of } 200 = n$$

$$\frac{20 = n}{100 = 200} \qquad \frac{20 \times 2 = n}{100 \times 2 = 200} \qquad n = 40$$

Example C:

$$n\% \text{ of } 80 = 20$$

$$\frac{20}{80} = \frac{n}{100}$$

$$\frac{20 \times 5/4}{80 \times 5/4} = \frac{n}{100}$$

$$n = 25$$

Example D:

$$20\% \text{ of } n = 16$$

$$\frac{20}{100} = \frac{16}{n} \qquad \frac{20 \times 4/5 = 16}{100 \times 4/5 = 0} \qquad n = 80$$

2. Make up 10 examples and solve by the equal ratio method.

Activity F (Grade 6)

Use the equation method in working these examples:

1. Study these examples.

Example A:

20% of 80 = n
.20 x 80 = n

Example B:

n x 80 = 16
n = 16/80 =

Example C:

.20 x n = 16
n = 16/.20

2. Make up 10 examples involving per cent and work by the equation method.

Activity G

Solve these examples by the ratio method and check by the equation method.

a. Find 4% of 60
b. 15% of 80 = n
c. 300% of 55 = n
d. 4 is ____% of 16
e. 12 is ____% of 40
f. 5% of n = 80
g. 60 = 120% of n
h. 600 = 150% of n